DAY THIRTY EIGHT

Mock Test 1

(Based on Complete Syllabus)

Instructions

1. This question paper contains of 30 Questions of Physics, divided into two Sections :
   Section A Objective Type Questions and **Section B** Numerical Type Questions.
2. Section A contains 20 Objective questions and all Questions are compulsory (**Marking Scheme** : Correct +4, Incorrect −1).
3. Section B contains 10 Numerical value questions out of which only 5 questions are to be attempted (**Marking Scheme** : Correct +4, Incorrect 0).

Section A : Objective Type Questions

1 A running man has half the kinetic energy of a boy of half his mass. The man speeds up by 1.0 ms^{-1} and then has the same kinetic energy as the boy. The original speed of the boy was

(a) 2.4 ms^{-1} (b) 9.6 ms^{-1} (c) 4.8 ms^{-1} (d) 7.2 ms^{-1}

2 The length of the string of a simple pendulum is measured with a meter scale, is found to be 92.0 cm, the radius of the bob plus the hook is measured with the help of vernier calliper to be 2.17 cm. Mark out the correct statement.

(a) Least count of meter scale is 0.1 cm
(b) Least count of vernier callipers is 0.01 cm
(c) Effective length of simple pendulum is 94.2 cm
(d) All of the above

3 Two bodies A and B of equal mass are suspended from two separate massless springs of spring constants k_1 and k_2 respectively. If the bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitudes of A to that of B is

(a) $\frac{k_1}{k_2}$ (b) $\sqrt{\frac{k_1}{k_2}}$ (c) $\frac{k_2}{k_1}$ (d) $\sqrt{\frac{k_2}{k_1}}$

4 The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is

(a) 11% (b) 21% (c) 42% (d) 10%

5 **Assertion** Thin prisms do not deviate light much.
Reason Thin prism have small angle A and hence, D_m (minimum deviation) is also very small as $D_m = [({}^1\mu_2 - 1) A]$, where ${}^1\mu_2$ is the refractive index of prism w.r.t. medium 1.
In the light of the above statements, choose the most appropriate answer from the options given below.

(a) Both A and R are correct and R is the correct explanation of A.
(b) Both A and R are correct but R is not the correct explanation of A.
(c) A is correct but R is not correct.
(d) A is not correct but R is correct.

6 Two bodies of different masses has been released from the top of tower. One is thrown in the horizontal direction while other is dropped, then which will reach the ground first?

(a) The body which has been thrown horizontally
(b) The body which has been dropped
(c) Both will reach the ground simultaneously
(d) Depends on the velocity with which the first body has been projected horizontally

7 In a hall, a person receives direct sound waves from a source 120 m away. He also receives waves from the same source which reach him after being reflected from the 25 m high ceiling at a point half-way between them. The two waves interfere constructively for wave lengths (in metre) of

(a) 10, 5, $\frac{5}{2}$, ... (b) 20, $\frac{20}{3}$, $\frac{20}{5}$, ...
(c) 30, 20, 10, ... (d) 35, 25, 15, ...

8 An AC source producing emf

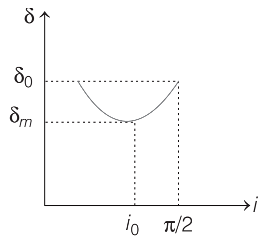
$$e = e_0 [\cos(100\pi \text{ s}^{-1})t + \cos(500\pi \text{ s}^{-1})t]$$

is connected in series with a capacitor and resistor. The steady state current in the circuit is found to be

$I = I_1 \cos[(100\pi\text{s}^{-1})t + \phi_1] + I_2 \cos[(500\pi \text{ s}^{-1})t + \phi_2]$

(a) $I_1 > I_2$
(b) $I_1 = I_2$
(c) $I_1 < I_2$
(d) The information is insufficient to find the relation between I_1 and I_2

9 In the diagram, a plot between δ (deviation) *versus* i (angle of incidence) for a triangular prism is given. From the observed plot, some conclusions can be drawn. Mark out the correct conclusions.

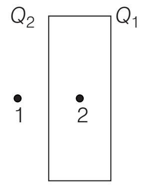


(a) The range of deviation for which two angles of incidence are possible for same deviation is $\delta_0 - \delta_m$
(b) The curve is unsymmetrical about i_0
(c) For a given δ, i is unique
(d) Both (a) and (b) are correct

10 A photon of 10.2 eV energy collides with hydrogen atom in ground state inelastically. After few microseconds one more photon of energy 15 eV collides with same hydrogen atom. Then what can be detected by a suitable detector?

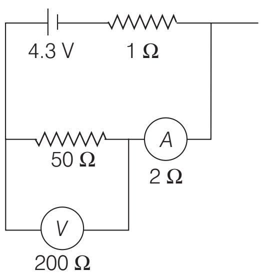
(a) 1 photon of 10.2 eV and an electron of energy 1.4 eV
(b) 2 photons of energy 10.2 eV
(c) 2 photons of energy 3.4 eV
(d) 1 photons of energy 3.4 eV and 1 electron of 1.4 eV

11 A non-conducting plate (infinite plane plate) is given a charge in such a way that Q_1 appears on one side and Q_2 on other side. The face area of plate is A. Find the electric field at points 1 and 2.



(a) $\frac{Q_1 + Q_2}{2\varepsilon_0 A}, \frac{Q_2 - Q_1}{2\varepsilon_0 A}$
(b) $\frac{Q_1 - Q_2}{2\varepsilon_0 A}, \frac{Q_1 + Q_2}{2\varepsilon_0 A}$
(c) $\frac{Q_1 + Q_2}{\varepsilon_0 A}, \frac{Q_2 - Q_1}{\varepsilon_0 A}$
(d) $\frac{Q_1 - Q_2}{\varepsilon_0 A}, \frac{Q_1 + Q_2}{\varepsilon_0 A}$

12 The emf and internal resistance of the battery as shown in figure are 4.3 V and 1 Ω respectively. The external resistance R is 50 Ω. The resistance of the voltmeter and ammeter are 200 Ω and 2 Ω respectively. Find the readings of the two meters.



(a) 0.1 A, 2 V (b) 0.1 A, 4 V
(c) 0. 4 A, 1 V (d) 0.4 A, 4 V

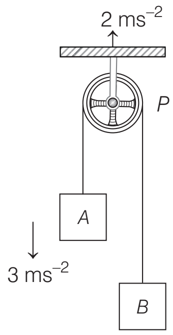
13 **Statement I** The rocket works on the principle of conservation of linear momentum.

Statement II Whenever there is the change in momentum of one body, the same change occurs in the momentum of the second body of the same system (having two bodies only) but in opposite direction.

In the light of the above statements, choose the most appropriate answer from the options given below.

(a) Statement I is true but Statement II is false.
(b) Both Statement I and Statement II are true.
(c) Both Statement I and Statement II are false.
(d) Statement I is false but Statement II is true.

14 All the accelerations as shown in figure are with respect to ground, find acceleration of B.



(a) 3 ms^{-2}, upward (b) 5 ms^{-2}, upward
(c) 3 ms^{-2}, downward (d) None of these

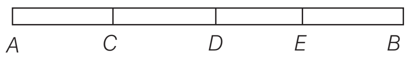
15 Light is incident at an angle α on one planer end of a transparent cylindrical rod of refractive index n. The least value of n for which the light entering the rod will not emerge from the curved surface of rod, irrespective of value of α is

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

16 A galvanometer has resistance $100\,\Omega$ and it requires current $100\,\mu$A for full scale deflection. A resistor $0.1\,\Omega$ is connected to make it ammeter. The smallest current in circuit to produce the full scale deflection is

(a) 1000.1 mA (b) 1.1 mA (c) 10.1 mA (d) 100.1 mA

17 A rod AB of uniform cross-section consists of four section AC, CD, DE and EB of different metals with thermal conductivities K, (0.8) K, (1.2) K and (1.50) K, respectively. Their lengths are respectively L, (1.2) L, (1.5) L and (0.6) L. They are joined rigidly in succession at C, D and E to form the rod AB. The end A is maintained at 100°C and the end B is maintained at 0°C. The steady state temperatures of the joints C, D and E are respectively T_C, T_D and T_E. Column I lists the temperature differences $(T_A - T_C)$, $(T_C - T_D)$, $(T_D - T_E)$ and $(T_E - T_B)$ in the four sections and column II their values jumbled up. Match each item in column I with its correct value in column II.



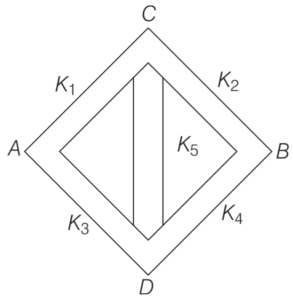
	Column I		Column II
A.	$(T_A - T_C)$	1.	9.6
B.	$(T_C - T_D)$	2.	30.1
C.	$(T_D - T_E)$	3.	24.1
D	$(T_E - T_B)$	4.	36.2

	A	B	C	D
(a)	3	4	2	1
(b)	1	2	4	3
(c)	3	4	1	2
(d)	3	2	1	4

18 A hole is bored along the diameter of the earth and a particle is dropped into it. If R is the radius of the earth and g is the acceleration due to gravity at the surface of the earth, then the time period of oscillation of the particle is

(a) $2\pi\sqrt{\frac{R}{g}}$ (b) $2\pi\sqrt{\frac{R}{2g}}$ (c) $2\pi\sqrt{\frac{2R}{g}}$ (d) $2\pi\sqrt{\frac{R}{3g}}$

19 Five rods of same dimensions are arranged as shown in the figure. They have thermal conductivities K_1, K_2, K_3, K_4 and K_5. When the points A and B are maintained at different temperatures, no heat flows through the central rod if



(a) $K_1 K_4 = K_2 K_3$
(b) $K_1 = K_4$ and $K_2 = K_3$
(c) $\frac{K_1}{K_4} = \frac{K_2}{K_3}$
(d) $K_1 K_2 = K_3 K_4$

20 A body dropped from a height H reaches the ground with a speed of $1.2\sqrt{gH}$. Calculate the work done by air-friction.

(a) 2.8 mgH (b) -1.3 mgH
(c) 1.3 mgH (d) $-$ 0.28 mgH

Section B : Numerical Type Questions

21 A raft of wood of density 600 kgm^{-3} and mass 120 kg floats in water. How much weight (in kg) can be put on the raft to make it just sink?

22 A 'double star' is a composite system of two stars rotating about their centre of mass under their mutual gravitational attraction. Let us consider such a 'double star' which has two stars of masses m and $2m$ at separation l. If T is the time period of rotation about their centre of mass, is found to be $2\pi\sqrt{\frac{l^3}{x\,Gm}}$, then the value of x is... .

23 A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the SONAR with a speed of 360 km/h. What is the frequency of sound (in kHz) reflected by the submarine? Take the speed of sound in water to be 1450 m/s.

24 If electric potential due to some charge distribution is given by $V = 3/r^2$, where **r** is radial distance, then electric field at (1, 1, 1) is found to be $\frac{2}{\sqrt{n}}(\hat{i} + \hat{j} + \hat{k})$, then the value of n is ....

25 Three dielectric slabs of thickness $d/4$, $d/7$ and $d/2$ having dielectric constants 2, 8/7 and 4 respectively are inserted between the plates of a parallel plate capacitor having plate separation d and plate area A. If the net capacitance of the system is equal to $\frac{p\varepsilon_0 A}{75\,d}$, then the value of p is ....

26 An electron of hydrogen atom is considered to be revolving around the proton in the circular orbit of radius $\frac{h^2}{4\pi^2 me^2}$ with velocity $\frac{2\pi e^2}{h}$. The equivalent current due to circulating charge is found to be $\frac{x\,\pi^2 me^5}{h^3}$, then the value of x ....

27 A sky wave with a frequency 55 MHz is incident on D-region of earth's atmosphere at 45°. Find the angle of refraction in degree. (Electron density for D-regions is 400 electron/cm^3.)

28 A 20 g bullet pierces through plate of mass $m_1 = 1$ kg and then comes to rest inside a second plate of mass $m_2 = 2.98$ kg. It is found that the two plates, initially at rest, now move with equal velocities. Find the percentage loss in the initial velocity of bullet when it is between m_1 and m_2 (neglect any loss of material of the bodies, due to action of bullet).

29 A block of wood has a mass of 25 g. When a 5 g metal piece with a volume of 2 cm^3 is attached to the bottom of the block, the wood barely floats in water. What is the volume V (in cm^3) of the wood?

30 If the half-life of a radioactive substance is 40 days, then find the duration in days in which it will decay 75% of its initial amount.

ANSWERS

1. (c)	**2.** (c)	**3.** (d)	**4.** (d)	**5.** (a)	**6.** (c)	**7.** (b)	**8.** (c)	**9.** (d)	**10.** (a)
11. (a)	**12.** (b)	**13.** (b)	**14.** (d)	**15.** (b)	**16.** (d)	**17.** (a)	**18.** (a)	**19.** (a)	**20.** (d)
21. (80)	**22.** (3)	**23.** (45.93)	**24.** (3)	**25.** (284)	**26.** (4)	**27.** (45)	**28.** (20)	**29.** (28)	**30.** (80)

Hints and Explanations

1 $(KE)_{man} = \frac{1}{2}(KE)_{boy}$

$\Rightarrow \frac{1}{2}mv_m^2 = \frac{1}{2}\left(\frac{1}{2} \times \frac{m}{2} v_b^2\right)$

$\Rightarrow \quad v_m = \frac{v_b}{2}$ ...(i)

Further,

$\frac{1}{2}m(v_m + 1)^2 = \frac{1}{2}\left(\frac{m}{2}\right)v_b^2$

$\Rightarrow \quad v_m + 1 = \frac{v_b}{2}$ ...(ii)

From Eqs. (i) and (ii), we get

$v_b = 2(\sqrt{2} + 1) = 4.82\ ms^{-1}$

$v_m = \sqrt{2} + 1 = 2.41\ ms^{-1}$

2 Effective length of the simple pendulum is $(92.0 + 2.17)$ cm

$= 94.2$ cm after rounding off to 3 significant digits.

3 Maximum velocity $= a\omega = a\sqrt{\frac{k}{m}}$

Given that, $a_1\sqrt{\frac{k_1}{m}} = a_2\sqrt{\frac{k_2}{m}}$

$\Rightarrow \quad \frac{a_1}{a_2} = \sqrt{\frac{k_2}{k_1}}$

4 $\because T = 2\pi\sqrt{\frac{l}{g}}, T^2 = 4\pi^2\frac{l}{g},$

$l = \left(\frac{g}{4\pi^2}\right)T^2$

$\therefore$ % change $= x + y + \frac{xy}{100}$

Vaild only for two variables in terms of percentage.

$x \rightarrow$ % change in first variable

$x \rightarrow$ % change in second variable

% increase in length $= x + x + \frac{x^2}{100}$

$21 = 2x + \frac{x^2}{100}$

On solving, $x = 10\%$

[by cross-check method]

5 For thin prisms, angle of prisms **A** is small.

For small A, D_{min} (minimum deviation) is also small.

So, $\mu = \frac{\sin\left(\frac{A + D_{min}}{2}\right)}{\sin(A/2)}$ ...(i)

$\sin\left(\frac{A + D_{min}}{2}\right) \approx \frac{A + D_{min}}{2}$

$(\because \sin\theta \approx \theta$ for small $\theta)$

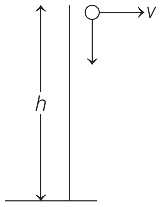
and $\sin\frac{A}{2} \approx \frac{A}{2}$

Using above approximations,

$$\mu = \frac{\frac{A + D_{min}}{2}}{A/2} \Rightarrow D_{min} = (\mu - 1)A$$

Hence, it can be seen that if **A** is small, D_{min} is also small.

6 Since, vertical displacement is same, as well as initial velocity in vertical downward direction is zero for both the bodies.



So, $h = \frac{g t_1^2}{2}$ (for horizontal throwing)

$h = \frac{g t_2^2}{2}$ (for dropping)

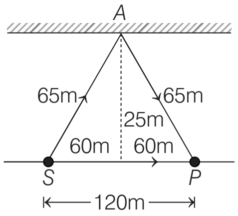
$\therefore \quad t_1 = t_2$

7 $\therefore$ Path difference,

$\Delta x = (SA + AP) - SP$

$= (65 + 65) - 120$

$\Rightarrow \Delta x = 10$ m



But at *A*, the wave suffers reflection at the surface of rigid/fixed end or denser medium. Hence, the wave must suffer an additional path change of $\frac{\lambda}{2}$ or a phase change of π.

$\Rightarrow$ Net path difference $= \left(10 - \frac{\lambda}{2}\right)$

For maxima (constructive interference),

Net path difference $= (2n)\frac{\lambda}{2}$;

where, $n = 0, 1, 2, \ldots$

$\Rightarrow \quad 10 - \frac{\lambda}{2} = 2n\left(\frac{\lambda}{2}\right)$;

where, $n = 0, 1, 2, \ldots$

$\Rightarrow$

$10 = (2n + 1)\frac{\lambda}{2}$; where, $n = 0, 1, 2, \ldots$

$\Rightarrow$

$\lambda = 20\,(2n + 1)$; where, $n = 0, 1, 2, \ldots$

or $\lambda = 20, \frac{20}{3}, \frac{20}{5}, \ldots$

8 $I_1 = \frac{e_0}{\sqrt{R^2 + \left(\frac{1}{\omega_1 C}\right)^2}} = \frac{e_0}{Z_1}$,

where, $\omega_1 = 100\,\pi$

$I_2 = \frac{e_0}{\sqrt{R^2 + \left(\frac{1}{\omega_2 C}\right)^2}} = \frac{e_0}{Z_2}$,

where, $\omega_2 = 500\,\pi$

So, $Z_1 > Z_2$, therefore $I_1 < I_2$.

10 When photon strickes the hydrogen atom, the photon is absorbed and H atom reaches in (*n*=2 state) or first excited state, emitting a photon of energy 10.2 eV. Ionisation energy of H-atom =13.6 eV, so the second photon of energy 15 eV will ionise the H atom and extra energy (15 -13.6)eV = **1.4 eV** will be retained by the electron. Thus finally we have one photon of energy 10.2 eV and one electron of energy 1.4eV.

11 At **1**; $E_1 = \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_1}{2\varepsilon_0} = \frac{Q_1 + Q_2}{2A\varepsilon_0}$

towards left

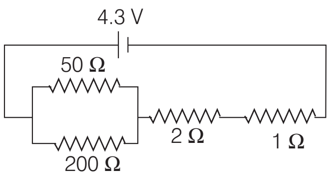
At 2; $E_2 = \frac{\sigma_2}{2\varepsilon_0} - \frac{\sigma_1}{2\varepsilon_0} = \frac{Q_2 - Q_1}{2A\varepsilon_0}$

towards right

where, $\sigma_2 = \frac{Q_2}{A}$ and $\sigma_1 = \frac{Q_1}{A}$

12 First of all draw the equivalent circuit diagram, current flowing through circuit

$= \frac{4.3}{(50 \parallel 200 + 2 + 1)} = 0.1$ A



Voltmeter reading $= 4.3 - 0.1 \times 3 = 4$ V

13 Since, in the rocket fuel is undergoing combustion, the gases produced in this process leave the body of the rocket with large velocity and produce upthrust to the rocket. Let us assume that the fuel is undergoing combustion at the constant rate, then rate of change of momentum of the rocket will be constant. Since, more and more fuel will be burnt the mass of rocket will go on decreasing, so it will lead to increase the velocity of the rocket more and more rapidly.

14 Consider downward direction as positive

$a_{AP} = -a_{BP}$

$a_{AG} = a_{AP} + a_{PG}$

$3 = a_{AP} - 2$

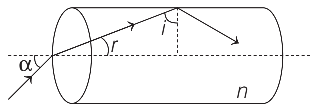
$\Rightarrow \quad a_{AP} = 5\ ms^{-2}$

$\therefore \quad a_{BG} = a_{BP} + a_{PG}$

$= -5 - 2 = -7\ ms^{-2}$

15 $\because r + i = 90°$

$\Rightarrow \quad i = 90° - r$



For ray not to emerge from curved surface,

$i > C$

$\Rightarrow \quad \sin i > \sin C$

$\Rightarrow \quad \sin(90° - r) > \sin C$

$\Rightarrow \quad \cos r > \sin C$

$\Rightarrow \quad \sqrt{1 - \sin^2 r} > \frac{1}{n}$

$\left[\because \sin C = \frac{1}{n}\right]$

$\Rightarrow \quad \frac{1 - \sin^2 i}{n^2} > \frac{1}{n^2}$

$\Rightarrow \quad 1 > \frac{1}{n^2}(1 + \sin^2 i)$

$\Rightarrow \quad n^2 > 1 + \sin^2 i \Rightarrow n > \sqrt{2}$

$[\because \sin i = 1 \text{ for } i = 90°]$

$\therefore$ Least value is $\sqrt{2}$.

16 $\frac{i_g}{i} = \frac{S}{S + G}$

$i = \frac{S + G}{S} \cdot i_g$

$= \frac{0.1 + 100}{0.1} \times 100 \times 10^{-6}$ A

$= 100.1 \times 10^{-3}$ A $= 100.1$ mA

17 A $\rightarrow$ 3; B $\rightarrow$ 4; C $\rightarrow$ 2; D $\rightarrow$ **1**

We have four sections, **AB**,**BC** , **CD** and **DE** with (**dQ / dt**) as the steady state thermal energy transmitted per second (**A** being the areas of cross-section)

$$\frac{dQ}{dt} = \frac{KA(100 - T_c)}{L}$$

$$= \frac{A(0.8)K(T_C - T_D)}{(1.5)L}$$

$$= \frac{(1.2)KA(T_D - T_E)}{(1.5)L} = \frac{(1.5)KA\,T_E}{(0.6)L}$$

These given

$(100 - T_C) = \left(\frac{8.0}{1.2}\right)(T_C - T_D)$

$= \left(\frac{1.2}{1.5}\right)(T_D - T_E) = \left(\frac{1.5}{0.6}\right)T_E$

$6(100 - T_C) = 4(T_C - T_D)$

$= (4.8)(T_D - T_E) = 15\,T_E$

Solving for the differences

$(100 - T_C), (T_C - T_D), (T_D - T_E)$ and T_E

remaining that the sum of these differences is 100, we obtain

$$(T_A - T_C) = 24.1,$$
$$(T_C - T_D) = 36.2$$
$$(T_D - T_E) = 30.1$$

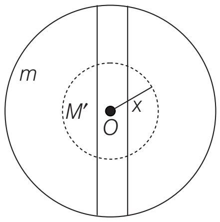
and $(T_E - T_B) = 9.6$

18 $F = -\dfrac{GM'm}{x^2}$

$$\Rightarrow F = -\frac{G\left(\frac{4}{3}\pi x^3\rho\right)m}{x^2}$$

$$= -\left(\frac{4}{3}\pi G\rho m\right)x$$

$$\Rightarrow x' = -\left(\frac{4}{3}\pi G\rho\right)x$$



Time period of oscillation,

$$T = 2\pi\sqrt{\frac{x}{|x'|}} = 2\pi\sqrt{\frac{3}{4\pi G\rho}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{R}{g}}$$

$$\left[\because g = \frac{Gm}{R^2} = \frac{4}{3}\pi R\rho G\right]$$

19 The arrangement of rods is analogous to the arrangement of resistances in a Wheatstone bridge balanced condition. Thus, no heat flows through the rod conductivity K_5, then

$$\frac{K_1}{K_3} = \frac{K_2}{K_4}$$

$$\Rightarrow \quad K_1K_4 = K_2K_3$$

20 The forces acting on the body are force of gravity and air-friction

According to work-energy theorem, total work done on the body = Gain in Kinetic energy

$$W = \frac{1}{2}mv^2 = \frac{1}{2}m\,(1.2\sqrt{gH})^2$$
$$= 0.72\ mgH$$

As work done by gravity,

$$W_1 = mgH$$

∴ Work done by friction,

$$W_2 = W - W_1$$
$$= 0.72\,mgH - mgH$$
$$= -0.28\,mgH$$

21 Volume of raft $= \dfrac{120}{600} = \dfrac{1}{5}\ m^3$

Fraction of volume inside water is

$$\frac{\rho_{wood}}{\rho_{water}} = \text{Relative density} = \frac{600}{1000} = \frac{3}{5}$$

So, fraction of volume outside water is

$$= \left(1 - \frac{3}{5}\right) = \frac{2}{5}$$

⇒ Volume outside water is,

$$V_{out} = \frac{2}{5}\times\frac{1}{5} = \frac{2}{25}\ m^3$$

When the raft just sinks, the additional upthrust is

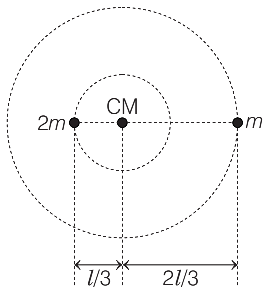
$$U = \frac{2}{25}\times 10^3\times g$$

The weight **m** put on the raft is

$$mg = \frac{2}{25}\times 10^3\times g$$

$$\therefore \quad m = 80\ \text{kg}$$

22 The system will revolve/rotate about an axis passing through the centre of mass of the combined system. Considering origin at the particle of mass $2m$, we have the centre of mass at a distance $l/3$ from $2l$ and $\dfrac{2l}{3}$ from **m**



The gravitational force of attraction between **2m** and **m** provides the necessary centripetal force to the mass to revolve in a circle of radius $\dfrac{2l}{3}$ for **m** or $\dfrac{l}{3}$ for **2m**.

$$\Rightarrow m\left(\frac{2l}{3}\right)\omega^2 = \frac{Gm\,(2m)}{l^2}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{3Gm}{l^3}} \Rightarrow T = 2\pi\sqrt{\frac{l^3}{3Gm}}$$

On comparing with

$$T = 2\pi\sqrt{\frac{l^3}{xGm}}\text{, we get}$$

$$x = 3$$

23 SONAR frequency,

$v_s = 40\ \text{kHz} = 40\times 10^3\text{Hz}$

Speed of enemy submarine

$$v_e = 360\ \text{km/h} = 360\times\frac{5}{18}\ \text{m/s}$$

$$= 100\ \text{m/s} \quad \left(\because 1\ \text{km/h} = \frac{5}{18}\ \text{m/s}\right)$$

Speed of sound in water = **1450** m/s

Apparent frequency received by the submarine,

$$v' = \left(\frac{v + v_e}{v}\right)v_s$$

$$= \left(\frac{1450 + 100}{1450}\right)\times 40\times 10^3$$

$$= 42.76\times 10^3\ \text{Hz}$$

Now, the reflected waves have a different frequency,

$$v'' = \left(\frac{v}{v - v_s}\right)v'$$

Here, v_s = **100** m/s is velocity of enemy submarine,

$$v'' = \left(\frac{1450}{1450 - 100}\right)\times 42.76\times 10^3$$

$$= 45.93\times 10^3\ \text{Hz} = 45.93\ \text{kHz}$$

24 As, $V = \dfrac{3}{r^2}$

$$\therefore\ E = -\left(\frac{dV}{dr}\right)r = -\frac{\partial}{\partial r}\left[\frac{3}{r^2}\right]r = \frac{6}{r^3}\,r$$

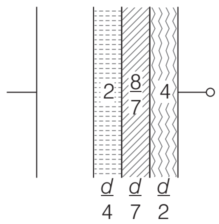
$$\Rightarrow \quad E = 6\,\frac{(\hat{i}+\hat{j}+\hat{k})}{(\sqrt{3})^3} = \frac{2}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$$

$$= \frac{2}{\sqrt{n}}(\hat{i}+\hat{j}+\hat{k})$$

(given)

$$\therefore \quad n = 3$$

25 Here, three slabs are in series



$$C_1 = \frac{\varepsilon_0 A}{\left(\frac{d/4}{2} + \frac{d/7}{8/7} + \frac{d/2}{4}\right)}$$

$$= \frac{\varepsilon_0 A}{\left(\frac{d}{8} + \frac{d}{8} + \frac{d}{8}\right)} = \frac{8\varepsilon_0 A}{3d}$$

$$C_2 = \frac{\varepsilon_0 A}{\left[d - \left(\frac{d}{4} + \frac{d}{7} + \frac{d}{2}\right)\right]} = \frac{28\,\varepsilon_0 A}{25\,d}$$

Now, $C_{eq} = C_1 + C_2 = \dfrac{284\,\varepsilon_0 A}{75\,d}$

$$= \frac{p\varepsilon_0 A}{75\,d} \qquad \text{(given)}$$

$$\therefore \quad p = 284$$

26 As, $T = \dfrac{2\pi r}{v} = \dfrac{2\pi\times h^2}{4\pi^2 me^2}\times\dfrac{h}{2\pi e^2}$

$$= \frac{h^3}{4\pi^2 me^4}$$

∴ Current, $I = \dfrac{e}{T} = \dfrac{4\pi^2 me^5}{h^3}$

$$= \frac{x\,\pi^2 me^5}{h^3} \quad \text{(given)}$$

$\therefore \quad x = 4$

27 $n_{eff} = n_0 \sqrt{1 - \left(\frac{80.5\ N}{\nu^2}\right)}$

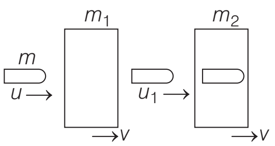
$$= 1\sqrt{1 - \frac{80.5 \times (400 \times 10^6)}{(55 \times 10^6)^2}} = 1$$

Also, $n_{eff} = \frac{\sin i}{\sin r} \Rightarrow \sin i = \sin r$

$\Rightarrow \quad r = i = 45°$

28 The situation is as shown in figure. Firstly take first sheet and bullet as the system,

$$mu = mu_1 + m_1 v$$



Now, take second sheet and bullet as the system,

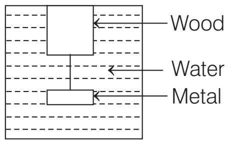
$$mu_1 = (m_1 + m_2)v$$

Solving this equation, we get

Percentage loss in

$$u = \frac{(u - u_1)}{u} \times 100\% = 20\%$$

29 Let volume of wood is $V\ cm^3$, then total volume of displaced water is $(V + 2)\ cm^3$, then for translational equilibrium, $(V + 2)\rho g = (25g + 5g)$



where all the quantities are in CGS unit and ρ is the density of water.

$\Rightarrow \quad (V + 2) \times 1 = 30$

$\therefore \quad V = 28\ cm^3$

30 Here, $N = N_0 \left(\frac{1}{2}\right)^{t/T}$

or $\quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T} \quad$ ...(i)

where, T is the half-life period and $\frac{N}{N_0}$ is fraction of atoms left after time t. Here, $T = 40$ days and $\frac{N}{N_0} = \frac{25}{100}$

$$= \frac{1}{4} = 0.25$$

Putting the values of T and $\frac{N}{N_0}$ in Eq. (i), we get

$$\frac{1}{4} = \left(\frac{1}{2}\right)^{t/40}$$

or $\quad \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{t/40}$

or $\quad \frac{t}{40} = 2 \quad$ or $\quad t = 80$ days